Correlation and blend in a mining productive system

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RESUME

The knowledge of the fluctuations in the feed of a mineralurgical or metallurgical mill is essential for estimating the recovery of the valuable products. Moreover, the knowledge of the average length of correlation is important for improving the controls and automation in the mill.

The model is based on a combination of the directional variograms in a zone of an open pit that yields the correlation in the output (ore plus waste) from the zone, and on a modified binomial model for determining the correlation and variation in the flow of ore. Standard statistics are then used to obtain the correlation and variation levels in the mill feed.

INTRODUCTION

In the mining industry, it is presently very hard to develop mathematical algorithms good enough for the solution of scheduling problems. In pratice, the miners tend to introduce heuristic algorithms; they then improve these in order to fulfill their changing needs. It would, however, be of a great scientific interest to develop a comprehensive mathematical model that would simplify computer needs on one part, and that would possibly evaluate various strategies on another part.

The actual situation derives from the fact that no model incorporates the segregation or blending processes at work in a transfer system even if these phenomena greatly influence the yield of mineralurgical or metallurgical operations : qualitative variations of the feed in a mill modify in a sensible way the recovery of valuable products.

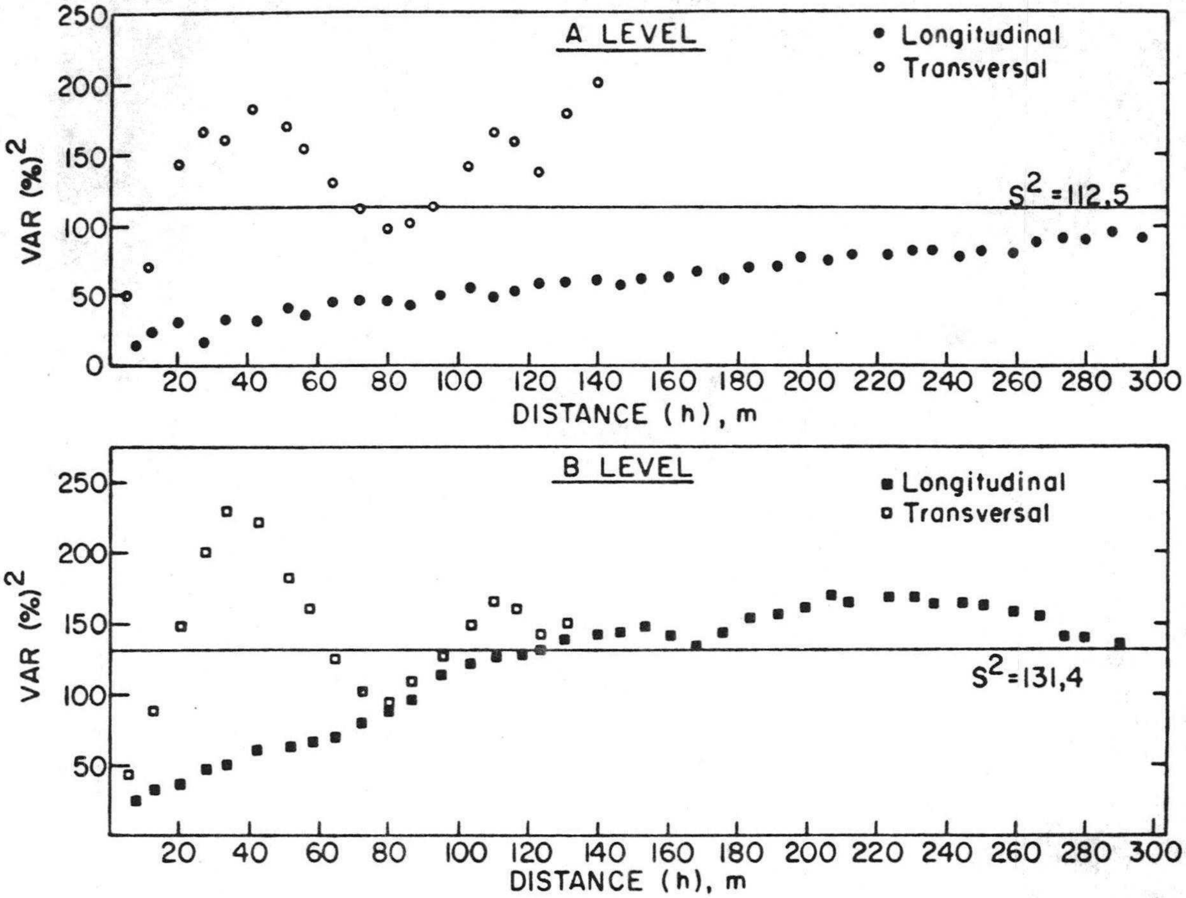
A mill is normally set at a fix point for grade and quantity feed. The incoming materials do not possess that stability, but rather fluctuate around those mean values. When too rich, the valuable minerals might not be all recovered, and there could be a penalty for too rich a concentrate grade in the contract with the smelter. When too poor, the minimum grade in the smelter contract might not be met with the concentrate.

This paper is concerned with the definition of intrinsic correlation of ore zones and also with its subsequent modifications by the decisional processes and the incurred blending between various zones of production. Ore bodies internal correlation is a well known characteristic for the geologists; many years of theoretical developments have yielded useful mathematical models to express it adequately. The decisional processes necessary to mine an ore zone modify the intrinsic correlation into a linear correlation in the flow of material taken out of the zone. The junction of materials from various locations into a single ore flow further modifies the characteristics of the individual flows.

CORRELATION IN AN OREBODY

It seldom happens, in an orebody, that fluctuations of a given mineral are of the same magnitude, whatever the orientation, and that no significant differences appear in the correlation from one or the other of the three dimensions. Such an orebody is said isotropic for the given variable.

Figure 1: Directional semi-..riograms of iron on two levels of a mi zone



In most cases, however, the orebodies are anisotropic, which means that there exists significant differences of the variation pattern between chosen directions (figure 1). That anisotropy induces quality control problems during the extraction, the planning engineer being forced to take into account variations of the mean, plus different variances and correlation patterns from level to level in the mine.

COMPUTING THE CORRELATION

Two functions are available for computing the similarity of neighboring samples : the variogram measures the level of fluctuations while the covariogram yields the degree of similitude between samples at varying intervals. These functions are shown on figure 2, with the relationship between them and the samples variance. It is not necessary to extend the discussion on a subject that is now formally accepted in mining; this presentation is based exclusively on the variogram.

It is however important to know the size and shape of the samples or their volume, in order to determine on what "support" the data are relying. Varying the sample lot size does modify the level of variations and consequently the knowledge of the deposit. This research has been carried on using blasthole chip sampling and the conclusions are entirely derived from the qualitative fluctuations of these raw data. It is quite realistic in practice because the final decisions for ore and waste selection are taken according to that level of information.

MODIFYING THE INTRINSIC CORRELATION

The materials extracted from a mine can be compared to the water flowing in a duct. The rock quantities being discrete instead of continuous like a fluid do not loose their individual characteristics very fast, so that it is quite easy to identify them on their way out of the mine. The major modification is their closeness in the production flow, because near by blocks

Figure 2: Relation between variogram and covariogram

VARIOGRAM: $\gamma(h) = \frac{1}{2(N-h)} \sum_{i=1}^{N-h} (x_i - x_{i+h})^2$

COVARIOGRAM: $C(h) = \frac{1}{N-h} \sum_{i=1}^{N-h} (x_i - \bar{x})(x_{i+h} - \bar{x})$

RELATION: $\gamma(h) = C(0) - C(h)$

WHERE: $\gamma(h)$ is the variogram at distance h
$C(h)$ is the covariogram at distance h
$C(0)$ is the variance (null distance)
N is the total number of samples
x_i is the value of the selected variable at point i
$\bar{x}$ is the mean of the samples

in situ cannot be extracted consecutively all the time. Consequently, the knowledge of the mining progression and of the underlying decision processes are required to estimate the linear ties when mining out the various materials.

The geometry of a deposit is often considered a key factor in defining the mining procedure to be used. A miner will open up an elongated zone longitudinally as illustrated on figure 3, while he will design the extration of a circular zone according to the grade projections and requirements.

A second item that influences the correlation in the production flow is the displacement of the loading unit in the zone : the average frequency and length of these moves must be determined for the period under study in order to stabilize the grade and quantities at the mill. Normally, decisions are taken at the beginning of the day shift and checked at the end of that shift; on some occasions, they are more closely adjusted to the requirements of changing conditions. When there is an excess of loading equipment capacity, decisions might imply a transfer of the production to a different area of the mine; otherwise, the working unit must move to another face of the zone.

In this presentation, the data of figure 1 have been used in order to model the extraction of the materials form the two levels of the zone. For this particular period, the mining has progressed -according to figure 3- about 80% longitudinally and 20% transversally, with variants going from 75%-25% in the first year to 90%-10% in the second year. The daily capacity of a loading unit is equivalent to 6 square blocks of 7,5 m side, or about 340 m^2 of a bench (12 m heigth); considering that 2 decision changes are taken daily, there will be 3 adjoining blocks mined out in succession before a move is made, let's say 60 m away.

The conjunction of the data on figure 1, the trends of mining and the decision modifications combine to yield figure 5 according to the equation of figure 4. Here is the explanation of each parameter : the values 0,50 - 0,50, 0,32- 0,68 and 0,38 - 0,62 represent the contribution of each level to the overall production; the constant proportions 0,75 - 0,25 illustrate the decisional process of mining three consecutive blocks on the average before modifying the position of the loading unit; and the variable fractions 0,75 -0,25, 0,90 - 0,10 and 0,80 - 0,20 are the relative influence of longitudinal and transversal semi-variograms on the resulting correlation in the mining of total materials.

CUT-OFF GRADE

Since Lane's publication on the subject in 1963, many studies have tried to ascertain the best cut-off grades for an operation and to elaborate a theory for that problem. This presentation will not try to expand on it; what is important here is the knowledge of the chosen cut-off grade for a given grade distribution.

Normally, the cut-off grade should fluctuate from zone to zone, according to characteristics of the orebody other than grade. As an example, the hardness of an ore yields a specific cost as far as grinding and crushing are concerned. Another item would be the fluctuations of the metal recovery from zone to zone that can modify the attractiveness of a particular area.

The sedimentary iron ore zone under study is in fact subdivided in three layers of different mean grade and dispersion. Because of deformations subsequent to their deposition, they are present on each horizontal slice of the zone in proportions modified from one level to the other. It is however possible to use a single cut-off grade for the three layers, and the value 46% Fe has been used to dissociate the direct shipping ore from the other materials.

Figure 3: Exemple of the prog. ...sion of mining in a zone much longer than its width. A,B, C: zones of varying grades.

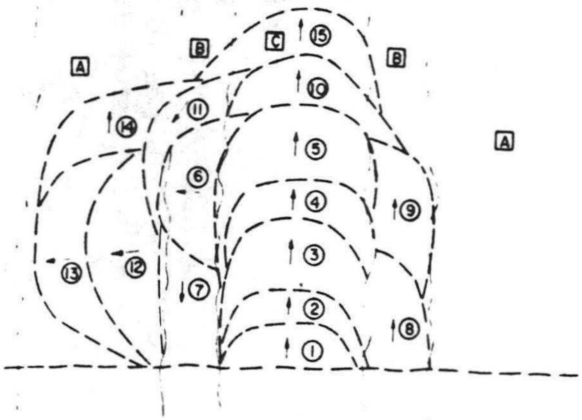


Figure 4 : Production semi-variograms computation formulae

$$\gamma_1(d) = 0{,}50\,(0{,}75(0{,}75\,\gamma_{AL}(h) + 0{,}25\,\gamma_{AL}(h+60)) + 0{,}25(0{,}75\,\gamma_{AT}(h) + 0{,}25\,\gamma_{AT}(h+60))) + 0{,}50(0{,}75(0{,}75\,\gamma_{BL}(h) + 0{,}25\,\gamma_{BL}(h+60)) + 0{,}25(0{,}75\,\gamma_{BT}(h) + 0{,}25\,\gamma_{BT}(h+60)))$$

$$\gamma_2(d) = 0.32\,(0.90(0.75\,\gamma_{AL}(h) + 0.25\,\gamma_{AL}(h+60)) + 0.10\,(0.75\,\gamma_{AT}(h) + 0.25\,\gamma_{AT}(h+60))) + 0{,}68(0{,}90(0{,}75\,\gamma_{BL}(h) + 0{,}25\,\gamma_{BL}(h+60)) + 0.10(0.75\,\gamma_{BT}(h) + 0.25\,\gamma_{BT}(h+60)))$$

$$\gamma_c(d) = 0{,}38(0{,}80(0{,}75\,\gamma_{AL}(h) + 0{,}25\,\gamma_{AL}(h+60)) + 0{,}20(0{,}75\,\gamma_{AT}(h) + 0{,}25\,\gamma_{AT}(h+60))) + 0{,}62(0{,}80(0{,}75\,\gamma_{BL}(h) + 0{,}25\,\gamma_{BL}(h+60)) + 0{,}20(0{,}75\,\gamma_{BT}(h) + 0{,}25\,\gamma_{BT}(h+60)))$$

WHERE $\gamma(d)$: production semi-variogram at a distance d equivalent to the directional length h in the deposit;

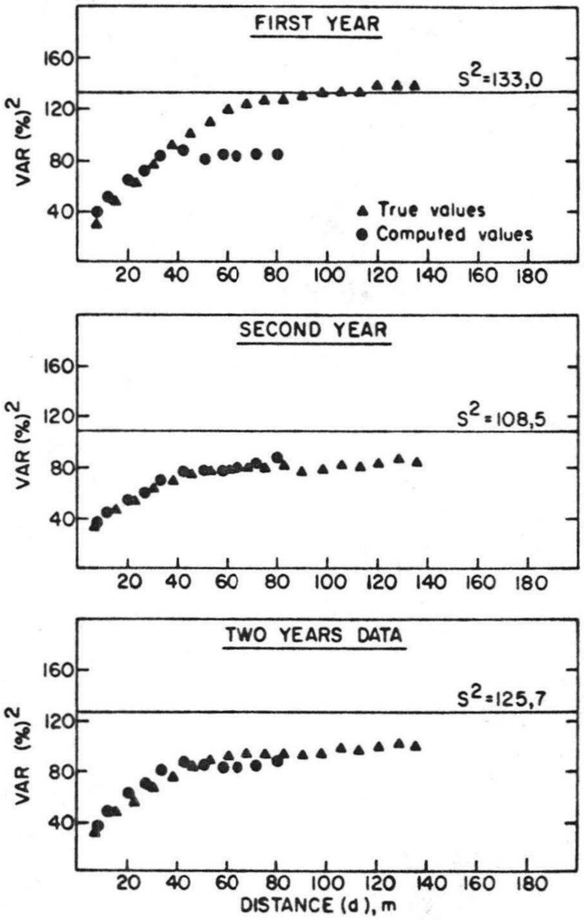
1,2,c : first, second year, two years together;

$\gamma(h)$: directional semi-variogram samples h meters apart;

L,T : longitudinal, transversal;

A, B : levels

Figure 5: Product... semi-variograms in the total materials going out of an operating zone.



DISTRIBUTIONS

The contribution of each layer to the overall extraction is different on each level; it is then preferable to use the layers distributions in order to reconstitute thereafter the mining of each year according to the relative contribution of those layers. Figure 6 illustrate the grade distributions of the individual layers of the zone.

It is then easy to apply the cut-off grade of 46% Fe on these distributions in order to dissociate ore from waste in each layer and define the relative proportion of ore in each one. Figure 7 indicates the major parameters in ore and waste (mean, variance, proportion) for each distribution.

CORRELATION IN THE ORE

A model as simple as the binomial can be used to express the correlation in the ore taken out of a zone in an open pit; the reason is that ore and waste must be mined alternatively by the loading unit, and also that decisions about the extraction of a mining area are centralized and appear random when seen from the given zone. A few hypothesis are however required to modify the model for expressing the correlation in ore itself.

Going back to figure 5, the maximal length of correlation can be estimated at 110-120 m for the data of the two years added together. This represents approximately 16 equivalent block lenghs of 7,5 m side. Looking then at figure 7, one can see that the proportion of ore and waste can be approximated respectively at 0,97 - 0,03, 0,82 - 0,18 and 0,35 - 0,65 in the three layers

Figure 6: Grade distributi[illegible] in three layers of a mining zone.

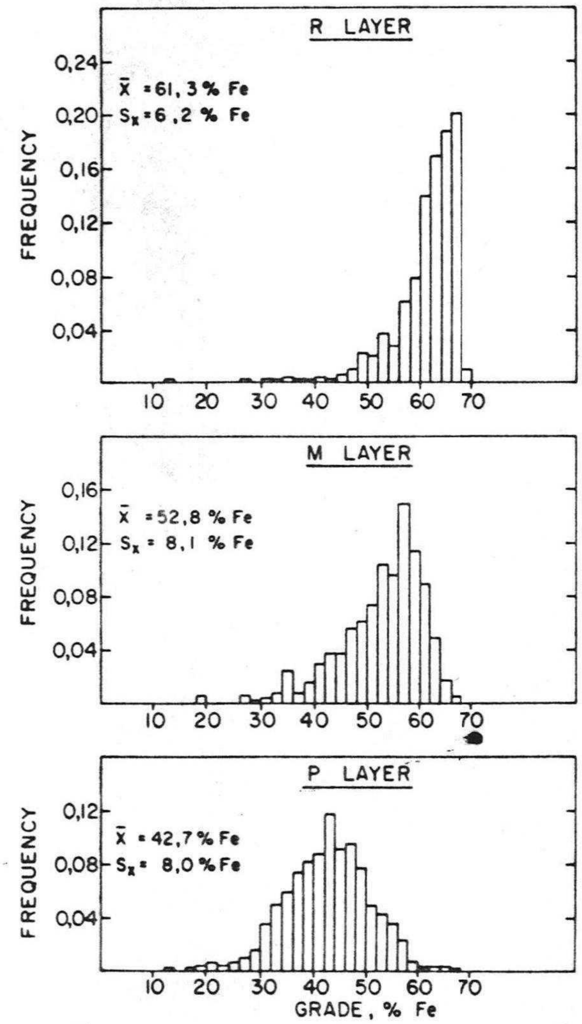
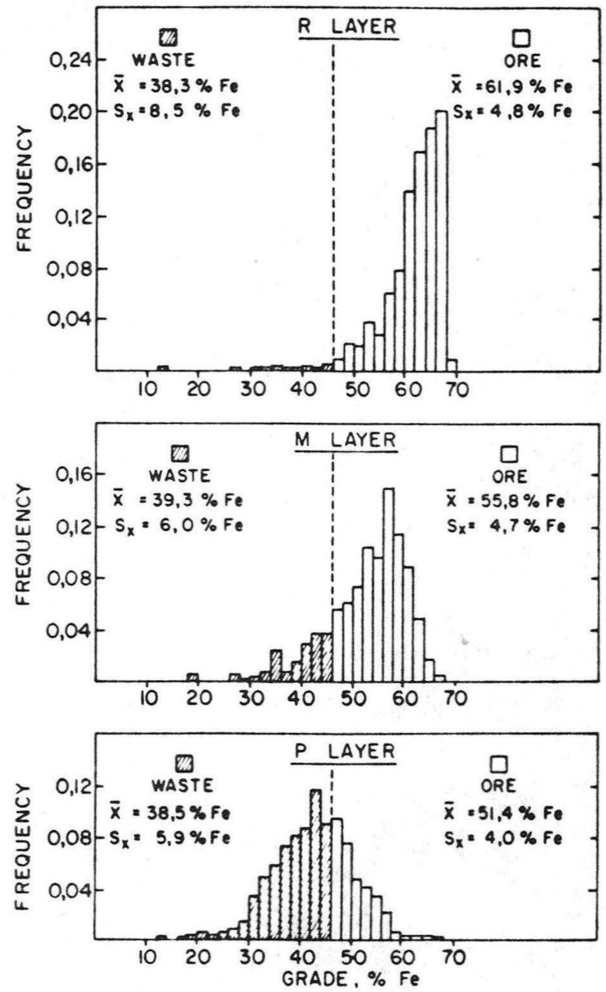


Figure 7: Grade [illegible]stributions in ore and waste of three layers of a mining zone.



of the zone under study. Using the expression of a single probability in the binomial model appearing on figure 8, one can compute the values in table 1; x_Z must be taken as the proportion of ore in the layer, $n = 16$ and ℓ varies from 0 to 16 and represents the number of ore units in the lot of 16 blocks of total materials.

Making the assumption that only uninterrupted ore sequences are correlated for a given length, the distribution of the 2^n probabilities of the binomial model must be modified. This hypothesis relies on the fact that the same loading unit is used for extracting ore and waste; when the unit is moved form ore to waste, the correlation stops in the ore sequence; when resuming the mining of ore, it is often in another area of the zone independant from the precedent lot.

According to the previous assumption, it is now necessary to recognize the probability of consecutive ore units in the lot of 16 blocks of total materials; figure 9 list the various possibilities of finding 13 consecutive ore lots. When 14 or 15 ore blocks are present, there are simultaneous observations of other ore units lengths. It is then necessary to separate the 2^n probabilities with equity; figure 10 shows the procedure for the attribution of the $2^{n-\ell}$ possibilities of measuring ℓ consecutive ore

LAYER / DISTANCE	R	M	P
0	0	0	0,001 015 345
1	0	0	0,000 546 724
2	0	0	0,000 294 390
3	0	0	0,000 158 518
4	0	0,000 000 001	0,000 085 356
5	0	0,000 000 002	0,000 045 961
6	0	0,000 000 011	0,000 024 748
7	0	0,000 000 049	0,000 013 326
8	0	0,000 000 225	0,000 007 175
9	0	0,000 001 026	0,000 003 864
10	0,000 000 001	0,000 004 675	0,000 002 080
11	0,000 000 017	0,000 021 297	0,000 001 120
12	0,000 000 562	0,000 097 019	0,000 000 603
13	0,000 018 172	0,000 441 975	0,000 000 325
14	0,000 587 553	0,002 013 441	0,000 000 175
15	0,018 997 536	0,009 172 343	0,000 000 094
16	0,614 253 654	0,041 785 119	0,000 000 051

FORMULA : $x_z^{\ell} (1 - x_z)^{n - \ell}$

TABLE 1 : Probability of single observations of length ℓ in the ore of each layer.

Figure 8: Single observation probability when using the binomial model

$$x_z^{\ell} (1 - x_z)^{n-\ell}$$

WHERE: x_z **represents the proportion of ore in zone z**

ℓ **is the number of ore units**

n **is the total number of materials units**

units. It is possible to combine the data of table I and figure 10 thereafter in order to obtain the values in table II; one can check the value of 0,02083 for ℓ = 13 in the layer R. Each value of table II is indicative of the relative contribution of a given length to the total variability of ore in the layer.

Figure 9 : Observations of 13 consecutive ore units in a lot of 16 units of material

i) When there are 15 units of ore in the lot

13 0 + 1W + 20

2 0 + 1W + 13 0

ii) When there are 14 ore units in the lot

13 0 + 2W + 1 0

13 0 + 1W + 1 0 + 1W

1 0 + 2W + 13 0

1 0 + 1W + 13 0 + 1W

1W + 13 + 1W + 1 0

1W + 1 0 + 1W + 13 0

iii) When 13 units of ore are present in the lot

13 0 + 3W

1W + 13 0 + 2W

2W + 13 0 + 1W

3W + 13 0

0 : ORE

W : WASTE

ℓ	LAYER R	LAYER M	LAYER P
1	0,03000	0,18000	0,65000
2	0,02910	0,14760	0,22753
3	0,02823	0,12103	0,07965
4	0,02738	0,09925	0,02790
5	0,02656	0,08138	0,00972
6	0,02576	0,06673	0,00339
7	0,02499	0,05472	0,00118
8	0,02424	0,04487	0,00041
9	0,02351	0,03680	0,00014
10	0,02281	0,03018	0,00005
11	0,02212	0,02476	0,00002
12	0,02146	0,02036	0,00001
13	0,02083	0,01698	0
14	0,02076	0,01521	0
15	0,03800	0,01834	0
16	0,61425	0,04179	0

TABLE II : Relative contribution of length ℓ to the variability of ore in each layer.

Figure [illegible] : Repartition of the $2^{n-\ell}$ possibilities of meeting consecutive units of ore in a lot of n units of materials

i) Taking $i = n - \ell$, the 2^i possibilities are

$\binom{i}{i} = 1$ possibility in a group of (n-1) ore units

$\binom{i}{i-1} = i$ possibilities in a group of (n-2) ore units

$\binom{i}{i-2} = \frac{i(i-1)}{2!}$ possibilities in a group of (n-3) ore units

.
.
.

$\binom{i}{1} + \binom{i}{0} = i+1$ possibilities in a group of ℓ ore units

ii) Example with $n = 16$ $\ell = 13$ $i = 3$ $2^i = 8$

- Repartition

$\binom{3}{3} = 1$ possibility in a group of 15 ore unit

$\binom{3}{2} = 3$ possibilities in a group of 14 ore units

$\binom{3}{1} + \binom{3}{0} = 4$ possibilities in a group of 13 ore units

- Individual probabilities of lengths 13, 14, 15 (Table I, R LAYER)
- Final Probability (ℓ = 13, R LAYER) :

1 0,018 997 536 + 3 0,000 587 553 + 4 0,000 018172 = 0,02083

It is now essential to define the relative contribution of each layer to the production of finite periods (table III) in order to compute the relative contribution of a given length to the total variability of ore during that span of time (table IV). A standardized semivariogram is then available for each period by summing up the values in table IV (table V); multiplying these last results by the variance of the mined ore blocks yields the semivariograms for the product during the period (table VI). The variance for the period is given on the last line of table VI; these values are representing semivariograms following the assumptions that consecutive ore blocks are correlated and that the behavior of the modified binomial model expresses the degree of correlation between these.

EFFECT OF SCHEDULING

The extraction decisions influence the correlation in the ore in the same way than total materials, in the sense that the handling equipment is moved 60 m further away from a given block or about 8 blocks apart twice a day on average. It is possible to compute the semivariograms of figure 11 from the data of table VI by the use of this simple equation :

$\gamma(d) = 0,75\ \gamma(\ell) + 0,25\ \gamma(\ell + 8)$

where $\gamma(d)$, $\gamma(\ell)$ are semivariograms and ℓ is the distance given in the first column of table VI.

PERIOD \ LAYER	R	M	P
FIRST YEAR	0,550	0,250	0,200
SECOND YEAR	0,175	0,100	0,725
TWO YEARS COMBINED	0,300	0,150	0,550

TABLE III : Relative contribution of each layer to the total production from the zone for the period.

DISTANCE \ PERIOD	FIRST YEAR	SECOND YEAR	FIRST PLUS SECOND YEAR
1	0,19150	0,49450	0,39350
2	0,09841	0,18481	0,15601
3	0,06171	0,07479	0,07043
4	0,04545	0,03494	0,03845
5	0,03690	0,01983	0,02552
6	0,03153	0,01364	0,01960
7	0,02766	0,01070	0,01635
8	0,02463	0,00903	0,01423
9	0,02216	0,00790	0,01265
10	0,02010	0,00705	0,01140
11	0,01836	0,00636	0,01036
12	0,01690	0,00580	0,00950
13	0,01570	0,00534	0,00880
14	0,01522	0,00515	0,00851
15	0,02549	0,00848	0,01451
16	0,34829	0,11167	0,19054

TABLE IV : Relative contribution of each distance to the variability of ore for different periods.

INFLUENCE OF MECHANICAL BREAKDOWNS

In the strongly mineralized areas like the R layer, most breakdowns occur when the loading unit is extracting ore; since the operating decisions are usually taken by assessing that the piece of equipment is available for mining ore, the pressure is strong for it to resume production as fast as possible after a breakdown and operate anew in ore. The blocks such mined when resuming production are often uncorrelated with the previous ones, so that the variability at a given distance is greater than the one computed by the binomial model. In other areas less mineralized, the breakdowns modify the observed sequences of ore mined, increasing de facto the correlation between ore blocks a given distance apart in the binomial model.

The mechanical availability of the loading equipment used in the zone has been around 75% during the first year and near to 85% in the second year. Most mining has been undertaken in the layers R and M during the first year and principally in the layer P during the second one. Consequently, it is possible to obtain figure 12

DISTANCE \ PERIOD	FIRST YEAR	SECOND YEAR	FIRST PLUS SECOND YEAR
1	0,19150	0,49450	0,39350
2	0,28991	0,67931	0,54951
3	0,35163	0,75410	0,61994
4	0,39708	0,78905	0,65839
5	0,43397	0,80888	0,68391
6	0,46550	0,82252	0,70351
7	0,49316	0,83322	0,71987
8	0,51779	0,84224	0,73409
9	0,53995	0,85014	0,74674
10	0,56005	0,85719	0,75814
11	0,57841	0,86355	0,76850
12	0,59531	0,86935	0,77800
13	0,61101	0,87469	0,78680
14	0,62623	0,87984	0,79531
15	0,65172	0,88833	0,80946
16	1,00000	1,00000	1,00000

TABLE V : Cumulative contribution of each distance to the variability of ore during each period, equivalent to standardized semivariograms.

DISTANCE \ PERIOD	FIRST YEAR	SECOND YEAR	FIRST PLUS SECOND YEAR
1	6,957	19,093	16,787
2	10,532	26,228	23,442
3	12,775	29,116	26,447
4	14,426	30,465	28,087
5	15,766	31,231	29,176
6	16,912	31,758	30,012
7	17,917	32,171	30,710
8	18,811	32,519	31,316
9	19,616	32,824	31,856
10	20,347	33,096	32,342
11	21,014	33,342	32,784
12	21,628	33,566	33,189
13	22,198	33,772	33,565
14	22,751	33,971	33,928
15	23,677	34,298	34,532
16	36,330	38,610	42,660

TABLE VI : Semivariograms of each period according to the modified binomial model.

Figure 11: Estimated producti emivariograms according to the binomial model.

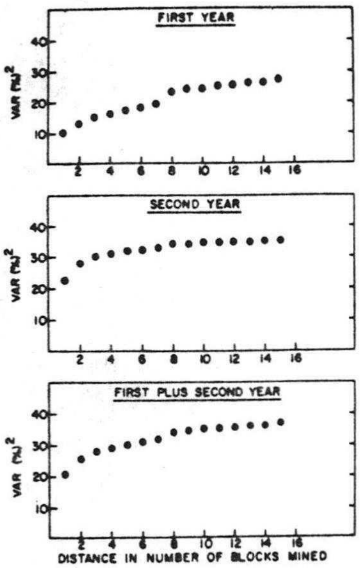
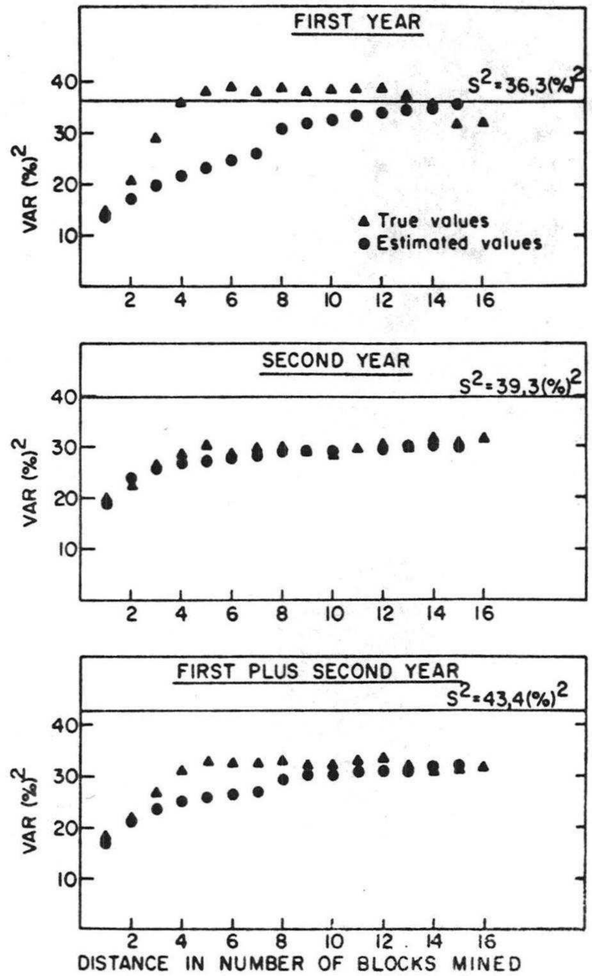


Figure 12: Semiv. grams of ore extracted from a zone .. an orebody.



from figure 11 by dividing the binomial model values by 0,75 the first year, by multiplying them by 0,85 the second year, and by ponderation of these two results (1/3, 2/3) for the data of the two years mixed together. The results are very closed to the true values (computed from the analysis of ore blocks production drilling chips) for the second year, but do not follow the real data during the first year. This is probably caused by transfers of the loading unit from ore of one layer to ore from another layer (almost uncorrelated blocks); the consequence would be to sharply reduce the correlation at a given distance in the ore production sequence.

CONCLUSION

The proposed binomial model has been shown to represent the correlation in an ore production sequence with some accuracy, and some weaknesses. It is however simple enough to be easily understood and implemented, and flexible enough to incorporate engineering criterias and decisional jugment. A more elegant solution is presently under development using disjunctive kriging, but it may take time before it is efficient enough to do the job.

When the dissociation of ore and waste takes place in an orebody, or when selecting ore blocks in an underground operation, it is not easy to find out the correlation in the ore mining sequence. This knowledge (or prevision) is however important for the characterization of these parameters elsewhere in the mining productive system, be it for combining flows, for blending piles or silos, for various handlings. This can make it possible thereafter to estimate the ideal optimal yield of the mine-mill complex, and to determine the closeness feasible solutions in practice.

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